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GEOMETRY.

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132. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics, Boys' High School, Reading, Pa.

To draw a circle to cut two given circles orthogonally.

I. Solution by VIRGINIA CRAIG, Senior Classical Student, Drury College, Springfield, Mo.

Let a and b be the radii of circles with centers P and Q, $a+b+c$ be the distance between P and Q, and the line joining P and Q cut the circles at E and F. Divide the line c in the ratio $c+2b : c+2a$, and let the point of division be O. Construct a semi-circle on OP as diameter, said semi-circle intersecting circle whose center is P at L. With O as center, OL as radius, describe a circle intersecting circle whose center is Q at M. This described circle will cut the two given circles orthogonally.

Proof. OL is perpendicular to PL.

$\therefore$ Circles with centers O and P intersect orthogonally.

$OE+OF=c$ and $OE : OF=c+2b : c+2a$.

$$\therefore OE=\frac{c(c+2b)}{2(a+b+c)},\ OF=\frac{c(c+2a)}{2(a+b+c)}.$$

$$OL^2=OE(OE+2EP)=\frac{c(c+2b)}{2(a+b+c)}\left(\frac{c(c+2b)}{2(a+b+c)}+2a\right)$$

$$=\frac{c(c+2a)}{2(a+b+c)}\left(\frac{c(c+2a)}{2(a+b+c)}+2b\right)=OF(OF+2FQ).$$

Let OX be the tangent from O to circle with center Q. Then $OX^2=OF(OF+2FQ)$. But $OF(OF+2FQ)=OL^2=OM^2$.

$\therefore$ $OX=OM$, and as M is on circumference of circle with center Q, OX and OM coincide.

$\therefore$ OM is tangent to circle with center Q and perpendicular to MQ.

$\therefore$ Circles with centers O and Q intersect orthogonally.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; J. W. YOUNG, Fellow and Assistant in Mathematics, Ohio State University, Columbus, O.; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.; M. E. GRABER, Heidelberg University, Tiffin, O.; and J. SCHEFFER, A. M., Hagerstown, Md.

Any circle with center on the radical axis and radius equal to the tangent will evidently satisfy the conditions of the problem. The problem, then, resolves itself into the construction of the radical axis of two circles.

If the circles intersect, the radical axis is the common chord. If they do not intersect, draw any other circle cutting the given circles and draw their common chords. Their point of intersection is the radical center of the three circles. The radical axis required must pass through this point and must be perpendicular to the line joining the centers of the given circles. This offers no difficulty and the problem is solved.

Also solved by ***ELMER SCHUYLER, HALLET E. McCLINTOCK;*** and ***CHARLES C. CROSS.*** Professor Young furnished a neat diagram to accompany his solution of Problem 131.

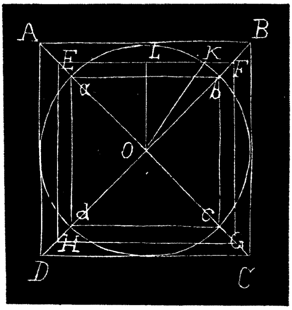
CALCULUS.

99. Proposed by L. C. WALKER, Associate Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

The axis of three equal right circular cylinders intersect at right angles. Find the volume of the solid common to all.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School Chester, Pa.

Let $x^2+z^2=a^2$, $y^2+z^2=a^2$, $x^2+y^2=a^2$ be the equations of the cylinders. The projection of the intersection of the first and second on the plane xy is a square.



From $z=0$, to $z=\frac{1}{2}a\sqrt{2}$ the square decreases from $ABCD$ to $abcd$. The area of the projection common to the three cylinders between $z=0$ and $z=\frac{1}{2}a\sqrt{2}$ is eight times the area $LObKL$. From $z=\frac{1}{2}a\sqrt{2}$ to $z=a$ the common volume is the same as the volume common to the first and second.

Let $\angle LOK=\theta$. Area $EFGH=4xy=4(a^2-z^2)$.

Area $LOK=\frac{1}{2}a.OL\sin\theta=\frac{1}{2}ay\sin\theta$.

Area $KOb=\frac{1}{2}a^2(\frac{1}{4}\pi-\theta)$.

$\tan\theta=LK/OK=x/y=\sqrt{(a^2-y^2)}/y=z/\sqrt{(a^2-z^2)}$;

$\therefore \sin\theta=z/a$. $\therefore$ area $LOK=\frac{1}{2}z\sqrt{(a^2-z^2)}$, area $KOb=\frac{1}{2}a^2(\frac{1}{4}\pi-\sin^{-1}z/a)$.

$$\therefore\ V=8\int_0^{\frac{1}{2}a\sqrt{2}}[z\sqrt{(a^2-z^2)}+a^2(\tfrac{1}{4}\pi-\sin^{-1}z/a)]dz+8\int_{\frac{1}{2}a\sqrt{2}}^{a}(a^2-z^2)dz,$$

$$=8a^3(2-\sqrt{2}).$$

Also solved by ***PROFS. ANDEREGG, SHERWOOD,*** and ***SCHMITT.***

100. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

What is the volume bounded by the surface generated by the circumference of a circle whose diameter is the hypotenuse of a right-angled triangle whose base is b and altitude a, the plane of the circle being perpendicular to the plane of the triangle, the triangle and circle being rigidly connected, and the triangle revolving about its altitude a as an axis?

Solution by F. ANDEREGG, A.M., Professor of Mathematics, Oberlin College, Oberlin, O., H. C. WHITAKER, Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; and the PROPOSER.

Let $AB=a$, $OA=b$, $AC=\sqrt{(a^2+b^2)}$; the coördinates of P and point in